Pergamon

Journal of Structural Geology, Vol. 16, No. 4, pp. 437 to 445, 1994
Elsevier Science Ltd
Printed in Great Britain
0191-8141/94 $6.00+0.00

Rotational quantities in homogeneous flow and the development of small-scale structure

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(*Received* 8 *January* 1993; *accepted in revised form* 28 *September* 1993)

Abstract—A summary discussion is given of attempts to quantify the degree of non-coaxiality of rock flow from observations of rock structure, and of some principles governing the sensitivity of structure to non-coaxiality. This is preceded by an extended review of relevant terminology, since some of the current confusion about rotational features in rocks stems from confusion about basic concepts and terminology from continuum mechanics. The kinematic vorticity number has been estimated for naturally deformed rocks from three localities. These yield vorticity numbers ranging from 0.35 to 0.9, corresponding to flows intermediate in character between pure and simple shearing.

INTRODUCTION

THIS paper reviews rotational properties of rock flow and finite deformation, and progress made in estimating the degree of non-coaxiality in natural flows. There is also some discussion of principles that govern whether and why developing rock structure is sensitive to the presence of a rotational component in the flow. These are increasingly popular subjects for research and part of a desirable long-term trend to view finite deformation structures in rocks as the residue of progressive deformation histories and associated progressive structural changes (Flinn 1962). The purpose of such work is to place greater constraints on models of rock flow history and associated deformation processes than can be provided by strain data alone. More extended explanations of rotational properties of flow than the summary given here, can be found in Passchier (1988) and Hanmer & Passchier (1991, pp. 5–22).

TERMINOLOGY

Strain

Strain is used in two different ways in contemporary structural geology. Both are correct usages, with respectable roots. In the usage of Ramsay & Huber (1983, p. 13) (also Nadai 1950, p. 112, Jaeger 1969, p. 33), strain is a change in configuration of particles of a body with components of distortion *and* rotation. A *rotational strain* is a strain with a non-zero rotational component. In the usage of Means (1976, p. 145) (also Nye 1957, p. 99, Malvern 1969, p. 125), a strain is just the distortional part of a general deformation. The rotation is a separate component. In this usage, there is no such thing as a 'rotational strain', only a rotational *deformation*. One practical justification for the narrower usage is that most so-called 'strain analyses' measure distortion only, not distortion-plus-rotation. Whichever usage one adopts, consistency within a given discussion is helpful. The meaning of strain employed here is the narrow one.

Partitioning

Most writers speak of strain partitioning when they mean a spatially uneven distribution of strain, over subregions or domains within some larger region. A special case of this usage is when one speaks of deformation *mechanism* partitioning, where different mechanisms dominate in, and develop distinct strains in, different subregions within a body. One can also speak of strain-rate, strain-path or vorticity partitioning, etc., again referring to a pattern of *spatial* heterogeneity in these quantities.

When strain is partitioned over a region, it is convenient to have a term for the part of the regional or *bulk strain* contributed by all domains of a given kind, for example domains characterized by dominance of different deformation mechanisms. I suggest the term *partial strain* for this purpose (Fig. 1). In particular circumstances, a partial strain may be, for example, a 'twinning strain' or a 'pressure solution strain' or an 'M-domain strain', etc.

Within any given kind of domain in a strain-partitioned material, there may be a characteristic local strain. I suggest that this be called a *local strain*. The relationship between bulk strains, local strains, and partial strains are further explained in Fig. 1. Bulk strains are weighted averages of local strains, and sums of partial strains. Similar terminology can be used for any partitioned quantity, for example strain-rate, dilation-rate or vorticity.

Superposition

Strain or deformation superposition refers to a *time* sequence of successive strains or deformations affecting one volume of material. An example is the superposition of a folding deformation upon an earlier, layer-parallel-

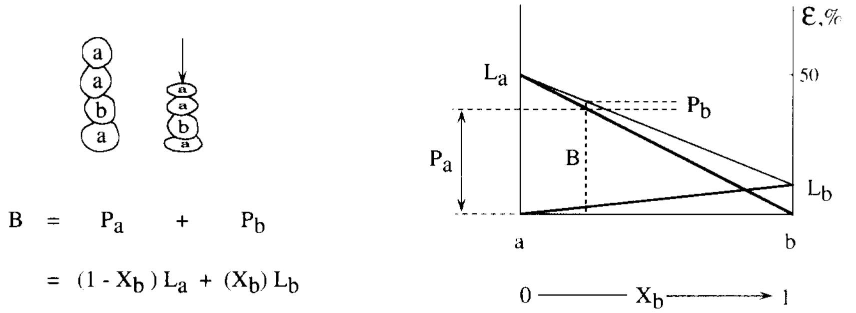


Fig. 1. Local, bulk and partial strains, illustrated using a line of grains of two different species, a and b, to represent two types of strain domains. The vertical line of grains shortens 50% where it crosses grains a and 10% where it crosses grains b. These are the *local strains* in each type of domain, L_a and L_b. The total or *bulk strain* of 40% of the whole line of grains is made up of two *partial strains* ($P_a = 38\%$, $P_b = 2\%$), which are the contributions to the bulk strain made, respectively, by all grains of type a and of type b. The partial strains depend in turn on the local strain in each type of grain and on the *length fraction* X (in the original state) of grains b (X_b) and a ($X_a = 1 - X_b$), as shown in the lower equation. The graph represents this equation, for all possible values of X_b. The two heavy lines show the partial strains, which vary linearly with X_b. The inclined lighter line represents the sum of the partial strains at any X_b, which gives the bulk strain B. The length of the vertical dashed line at $X_b = 0.25$ corresponds to the bulk strain for the case illustrated.

shortening deformation (e.g. Simon & Gray 1982). Where the deformations are large, superposition needs to be carried out mathematically by matrix multiplication (see Ramsay & Huber 1983, p. 292).

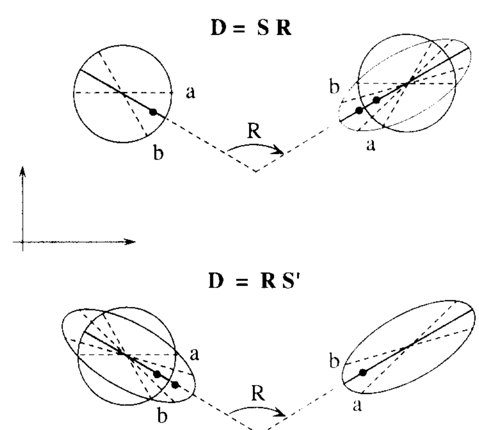
Factorization

Finite strain or deformation factorization is mathematical *decomposition* of these quantities into parts which can be recombined, by matrix multiplication, to yield the original quantity (Ramsay & Huber 1987, p. 638). Factorization can be carried out for a wide variety of purposes. Bell (1979) factorizes to separate a tectonic strain ellipse from a pre-tectonic fabric ellipse. Evans & Dunne (1991) factorize to define a proposed time sequence of tectonic events. De Paor (1986) factorizes to isolate a volume-change component. Oertel & Reymer (1992) factorize to separate a local strain perturbation from an average strain. All these are perfectly valid factorizations. In some cases the matrix multiplication corresponds to a superposition, but in others it does not. There are always an infinite number of mathematically valid factorizations of a strain or deformation, most of them useless.

Rotation

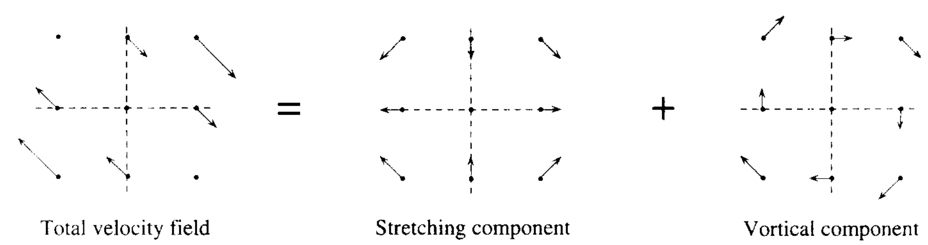
An important factorization for present purposes is the factorization of a finite deformation into components of rotation and strain. This can be done in two ways (Fig. 2). A strain component can be found that *pre-multiplies* the rotation. This is called the *left-stretch tensor*. Or a strain component can be found that *post-multiples* the rotation (the *right stretch tensor*). The components of the two stretch tensors are different, because they correspond, respectively, to performing the strain in the rotated or original states (Schwerdtner 1979) (Fig. 2). The rotation part has identical components, whichever decomposition is used. It represents the rigid-body rotation that would carry the orthogonal material lines in the principal directions of strain, from their orientations in the original state to their orientations in the deformed state. The rotational part of any finite deformation can also be defined as the *average* rotation of all material lines passing through a particle. This alternative definition exists because rotations associated with the strain component are symmetrically paired, so they cancel each other out (Fig. 2).

A simple, but too-often overlooked, rule is that ro-



$$\text{In both cases, } R_{ij} = \begin{bmatrix} \cos R & \sin R \\ -\sin R & \cos R \end{bmatrix}$$

Fig. 2. Alternative decompositions of the finite deformation **D**, and the meaning of the finite rotation **R**. In the upper decomposition, the order of matrix multiplication corresponds to performing the rigid-body rotation first and the distortional part of the deformation (the 'stretch') second, without further rotation of material lines in the principal directions of stretch. In the lower decomposition, the order of matrix multiplication corresponds to performing the stretch first, without rotating lines in the principal directions of stretch, and then performing the rigid-body rotation. The components of the rigid-body rotation **R** are identical for both decompositions, but the components of **S** and **S'** are different. Note that any generally-oriented line (a) is rotated by the stretch part of the deformation (relative to the principal axes of stretch) but that there is always another line (b) that is symmetrically disposed across the principal axes, that rotates by an equal amount but in the opposite sense. So all these rotations associated with the stretch add up to zero. The average rotation of all material lines is therefore equal to the rotation (R) of the principal axes of stretch, in the designated reference frame.



$$\begin{bmatrix} .3 & .3 \\ -.3 & -.3 \end{bmatrix} = \begin{bmatrix} .3 & 0 \\ 0 & -.3 \end{bmatrix} + \begin{bmatrix} 0 & .3 \\ -.3 & 0 \end{bmatrix}$$

Components of velocity gradient tensor **L** — Components of stretching tensor **s** — Components of vorticity tensor **W**

Fig. 3. Decomposition of the velocity field about a point into its stretching and vortical components. In this particular case the total field is a homogeneous simple shearing at 45° to the reference axes (dashed). Note that the velocity vectors in the top diagrams and the tensor components below them, *add* to yield the vectors shown for the total velocity field, and the components in the matrix of the tensor **L**. The numbers in the **L** matrix are related to the velocity variations from point to point (the velocity gradients) in the corresponding vector diagrams as follows. The upper left component (L_{11}) is 0.3 because the horizontal component of velocity (v_1) varies by +0.3 between any pair of particles separated by unit distance in the direction of the positive horizontal co-ordinate axis (x_1). (Assume unit distance for the distance between marked particles, horizontally and vertically; also assume velocity vectors drawn to the same scale numerically as the interparticle distances.) The lower left component (L_{21}) is −0.3 because this is the variation or gradient in v_2 for each unit distance in the positive direction along x_1. Similarly with the two components in the right column of the **L** matrix (L_{12} and L_{22} reading downward). These give the variation or gradients in v_1 and v_2 between particles separated by unit distance in the vertical (x_2) direction.

tations are reference-frame dependent. It means nothing to say that a rotation is zero or non-zero, unless a reference frame is also specified. A writer may assume that a reader will understand that he means zero rotation with respect to a *geographic* reference frame, but this should never be left unstated. There are many other features to which reference frames can be pinned, including bedding or foliation traces and axial planes or shear zone boundaries. Specification of a reference frame is equally important whether one is dealing with the rotational component of a deformation—a rigid-body rotation of a whole volume or area of material—or with the rotation of a particular material line. A reference frame *can* always be chosen that is fixed to material lines in the principal directions of finite strain, rotating with them in geographic space. Where this choice is made, any finite deformation becomes *irrotational*.

When reference frames are chosen with one axis fixed to internal structural features, like the boundaries of twin lamellae or larger-scale shear zones, the rotations of lines have been called *internal rotations* (Sander 1970, p. 38). We can extend this usage here, and use the term internal rotation as a general term for the rotation of an individual material line, or the average rotation of all material lines, relative to some reference frame other than the geographic one. But again, the particular internal reference frame must always be exactly specified. Reference frames can be pinned to any material line as above, or to any immaterial but identifiable line—such as the trace of a migrating fold axial plane.

The term *external rotation* can be used, again following Sander, for rotations relative to a geographic reference frame. But sometimes it may be convenient to use internal and external simply to distinguish reference frames on two different, nested scales—for example internal rotation of worm burrows relative to bedding on a fold limb and external rotation of the worm burrows relative to the axial plane of the fold.

Vorticity

Much of what has just been said about the rotational component of a deformation applies similarly to the rate-of-rotation quantity called the *vorticity*. The flow at any point in a continuum *at an instant* can be factorized into a pure rotating part called the vorticity, and a pure straining part called the stretching component (Fig. 3). These *add* vectorially to yield the total velocity field at the point and *add* as matrices to give the *velocity gradient tensor* (L) that describes the velocity field locally. (Readers unfamiliar with the velocity gradient tensor can find an introduction to it in Passchier (1988, pp. 323–325) and in Means (1990, p. 955). A few details are also explained here, in the caption of Fig. 3). Like the rotation, the vorticity is reference-frame dependent. In particular, it can be viewed from some relatively remote reference frame, like a geographic frame or a large fold-axial plane, or it can be viewed from the reference frame of the *stretching directions*—the principal directions of the instantaneous increment of strain (horizontal and vertical in Fig. 3). These have been called the *external* and the *internal vorticity*, respectively (Means *et al.* 1980).

When expressed as a single number, the magnitude of the vorticity (W) is *twice* the average angular velocity of all material lines through a given particle, or, equivalently, the sum of the angular velocities of any instantaneously orthogonal pair of material lines. The symmetrically paired angular velocities associated with the stretching component cancel each other out, and the residue is a set of angular velocities that are those of a rotating rigid body. The definition of vorticity magni-

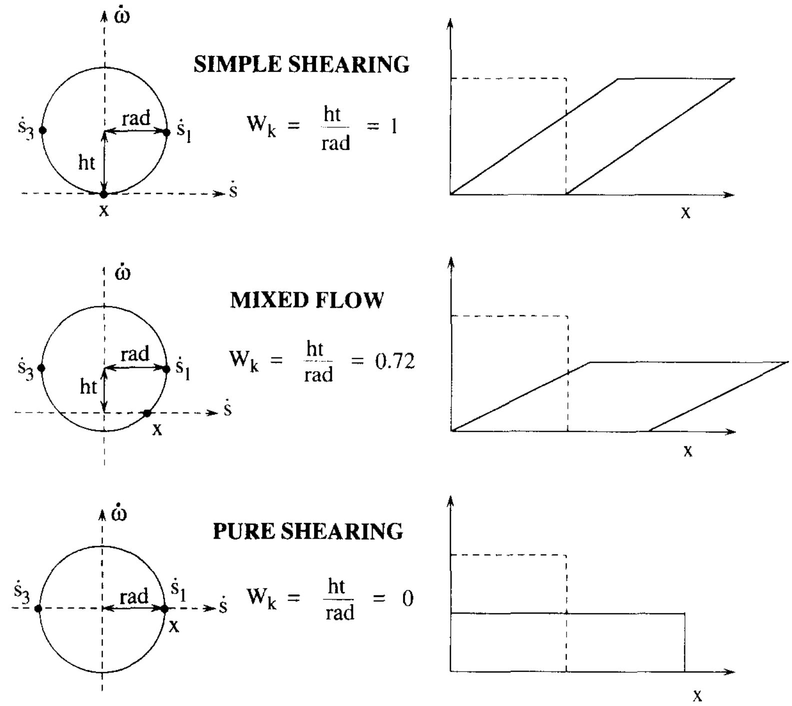


Fig. 4. Mohr circles for the velocity gradient tensor **L** and vorticity numbers W_k for simple shearing, pure shearing and a typical intermediate or 'mixed' flow. Diagrams on the right indicate finite deformations resulting from a steady flow of each type, maintained for a suitable time. The vorticity number for each flow type is equal to the ratio of the height (ht) of the center of the Mohr circle above the horizontal Mohr axis, to the radius (rad) of the circle. The height is the average angular velocity ($\dot{\omega}$) of all material lines (half the vorticity), and the radius is half the difference between the maximum and minimum extension rates or 'stretchings' $\dot{s}_1$ and $\dot{s}_3$.

tude as *twice* this rigid-body rotation-rate seems unnatural (Simpson & De Paor 1992), but it leads to a simple definition of the 'vorticity number' defined below. The units of vorticity are *radians* per unit time.

The choice of the instantaneous stretching directions for a reference frame is an important one because flows having vorticity in this frame are flows in which material rotates *through* the stretching directions. Simple shearing is an example. All such flows are said to be *non-coaxial*, because (at least two of) the principal directions or axes of incremental strain at each moment are not parallel to the principal directions of the finite strain so far accumulated. The response of rocks to this asymmetrical type of strain accumulation is of interest because it is expected to be common in high-strain zones of most kinds, including shear zones and the limbs of many folds.

Vorticity number

Vorticity alone is not a very significant quantity for structural development. The effect that vorticity has on strain-induced structures depends on the *ratio* of the vorticity magnitude to some measure of the concurrent rate of straining. A measure proposed by Truesdell (1953) is the ratio of the vorticity to the different between the principal stretchings $\dot{s}_1$ and $\dot{s}_3$. This is called the kinematic vorticity number (W_k), and it ranges from 0 for coaxial, pure shearing to 1 for simple shearing. Values greater than 1 are also possible (Ramberg 1975, Ramsay & Huber 1983, p. 233, Weijermars 1991) though they are expected to prevail only locally in flowing rock bodies, in eddy-like local concentrations of vorticity. The definition of W_k given above is suitable for two-dimensional flows without area-change, or for three-dimensional flows viewed in a plane of no area-change normal to the vorticity vector (the axis of rotation). More general definitions and discussion are given by Means *et al.* (1980), Passchier (1987) and Weijermars (1991).

Vorticity numbers can be understood graphically by drawing Mohr circles for the **L** tensor (see Means 1990, p. 955) for situations intermediate between pure and simple shearing (Fig. 4). These circles are graphs of angular velocity of material lines vs extension-rate, for lines in all orientations through a given point. The vorticity number for a flow is the ratio of the vertical distance of the center of the circle from the horizontal axis to the radius of the circle (Lister & Williams 1983).

Passchier (1988) has shown how this same relationship exists in Mohr space for a Mohr circle representing the finite deformation tensor. If a flow is steady and characterized by a vorticity number of 0.7, the finite deformation throughout the flow will also be represented by a Mohr circle that has a ratio of 0.7 between the distance of its center from the horizontal axis and its radius. Passchier calls this ratio the *mean vorticity number*. For steady flows, it is the true vorticity number that actually prevailed throughout the history. For non-steady flows with varying vorticity numbers, the mean vorticity number has no simple relationship to the history of W_k, but it may sometimes be a useful 'average' value anyway. The mean vorticity number, interpreted as above, requires a reference frame as in the right parts of Fig. 4, fixed

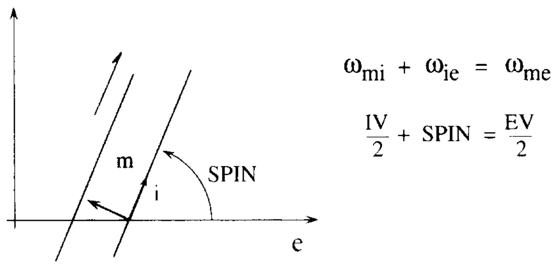


Fig. 5. A shear zone (shaded) with attached internal reference axes i rotating in an external reference frame e at an angular velocity defined as the spin or ω_{ie}. When this spin is added to the average angular velocity of material in the shear zone relative to axes i (ω_{mi}), the sum is the average angular velocity of the material with respect to the external frame (ω_{me}). This leads, via the definitions of internal vorticity (IV) and external vorticity (EV), to the second equation.

relative to a material line that was itself fixed relative to the stretching directions.

Spin

The spin (following Means *et al.* 1980) is the rate of rotation of the stretching directions relative to some external (often geographic) reference frame. There is an additive relationship between external vorticity, internal vorticity and spin (Fig. 5); the external vorticity can be factorized into a spin component and an internal vorticity component which add to give the external vorticity. The spin component is not expected to have any direct effect on structure, except where structures are sensitive to the direction of the gravitational acceleration. Mohr diagrams showing spin added to the internal vorticity raise or lower the circles relative to the horizontal axis.

The addition of spin to a flow can always erase its external vorticity, but no amount of added spin can remove non-coaxiality. This is represented by the Mohr diagrams of Fig. 6 for a steady simple shearing. The

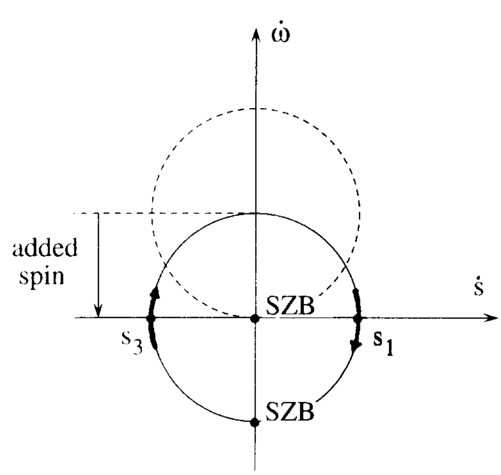


Fig. 6. Mohr circles for the **L** tensor, for a simple shearing with zero spin (dashed), and with just the right counterclockwise spin added to make the external vorticity zero (solid). In this condition material lines instantaneously under the stretching directions are not rotating in the external reference frame, but they *are* rotating clockwise relative to the stretching directions. This latter rotation, which makes the flow non-coaxial, is represented by the heavy arrows passing through the $\dot{s}_1$ and $\dot{s}_3$ points. Point SZB on both circles represents material lines parallel to the shear zone boundary, taken to be the x_1 direction.

internal vorticity essential for non-coaxiality is represented by the circumferential arrows indicating how points for particular material lines *circulate* in one direction around the circle, passing through the stretching direction points, and all points but the 6 o'clock point (labeled SZB). They circulate at twice the angular rate that the corresponding material lines are rotating in geographic space. Adding spin can raise or lower the circle, but never remove this circulation.

RECOGNITION OF NON-COAXIALITY FROM STRUCTURE

Recognition that rock fabrics can reflect non-coaxiality of flow goes back at least to Sander (1911). According to Sander, the Symmetry Principle requires that velocity or displacement fields possessing a single plane of symmetry should give rise to fabrics possessing the same single plane of symmetry. Thus, non-coaxial flows are expected to result in fabrics of lower symmetry than coaxial flows. Coaxial flows acting on initially isotropic rocks should give fabrics with three or more planes of symmetry. Law *et al.* (1984) and Schmid & Casey (1986), for example, use these rules to interpret quartz fabrics and there are many similar interpretations in the literature. The Symmetry Principle is a powerful intuitive aid to fabric interpretation, but no one yet has found a way to make it yield quantitative information on the degree of non-coaxiality in flows intermediate between pure and simple shearing. In addition, there are two obstacles to using fabric asymmetry as an indicator of non-coaxiality. Where the initial fabric is anisotropic, a low-symmetry deformed fabric can be expected, even from a high-symmetry, coaxial flow pattern (Paterson & Weiss 1961). Second, two perfectly coaxial, non-spinning flows, if superposed with non-parallel stretching directions, can give a total, rotational deformation in the geographic reference frame (Flinn 1978), with asymmetric fabric features mimicking the product of a single deformation accumulated non-coaxially. This is closely related to the fact pointed out by Ramsay (1962), that oblique superposition of successive strains is always expected to lower the symmetry of whatever internal rock structures are influenced by both strains.

Incremental strain indicators are particularly helpful for recognizing the products of non-coaxial flow. These were used in the classic papers by Durney & Ramsay (1973), Wickham (1973) and Berthé *et al.* (1979), where crystal fibers were taken to indicate the direction of late extension, and non-coaxiality was inferred from non-parallelism of these late fibers with earlier ones (Wickham, Durney and Ramsay) or with an *S* foliation believed to represent the local, total extension direction (Berthé *et al.*).

Another qualitative indicator of non-coaxiality was employed by Beach (1979), who observed asymmetric orientations of directions of no finite elongation in belemnite populations, when the orientations were measured clockwise and then anticlockwise from the

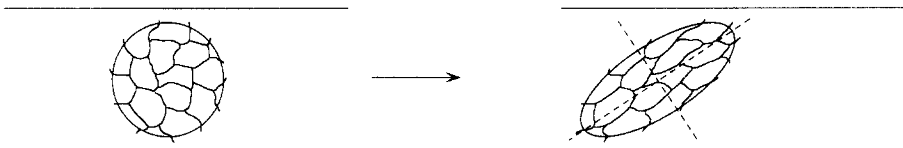


Fig. 7. An example of apparent violation of the Symmetry Principle. A grain-shape foliation (right) with the full orthorhombic symmetry of the strain ellipsoid, develops during a progressive simple shearing, in which the symmetry of the velocity and displacement fields is not orthorhombic, but monoclinic (in a reference frame fixed to the shearing direction). The dashed lines represent planes of fabric symmetry that are absent in the velocity and displacement fields. This behavior requires ideally passive behavior of the initial grain boundary array (left) throughout the progressive deformation.

direction of maximum finite elongation. This is not expected if the belemnites behave like ideal passive markers, even in a non-coaxial situation. Beach attributes the asymmetry in his rocks to a combination of non-ideal behavior of the belemnites (lengthening but not shortening with the matrix) and non-coaxiality of the flow.

Schrader (1988) describes how an asymmetrical arrangement of striations on pebble surfaces in deformed conglomerates may be used to recognize non-coaxiality. The striations are interpreted as direct indicators of displacement directions of matrix particles relative to the pebble surface.

The first quantitative estimate of the vorticity number that I know of was obtained by Passchier & Urai (1988), using inferred rotations for a vein-set and for bedding in a slate outcrop, and a stretch value for the (boudinaged) vein set. The basis for this and related methods is discussed in Passchier (1990). The value of the mean vorticity number obtained was 0.8 ± 0.1, suggesting a flow closer to simple shearing than to pure shearing. The next estimate was made by Vissers (1989) using the shapes and rotations of garnets in a mylonitic quartzite. A ratio of pure shearing to simple shearing of 0.65 was obtained, which translates into a vorticity number of about 0.6.

More recently, Wallis (1992) has made an estimate of the mean vorticity number in metacherts, using radiolarian shapes to define the finite strain, and the angular distribution of veins with various stretching histories to define the degree of non-coaxiality. Mean vorticity numbers between about 0.5 and 0.7 were obtained. An independent estimate of mean vorticity number in the same rocks was made, assuming that the central part of the quartz *c*-axis girdle is parallel to a principal plane of strain (see Wallis 1992 for details). This estimate yielded a vorticity number in the range 0.35–0.6, consistent with the estimated based on radiolaria and veins.

SENSITIVITY OF STRUCTURE TO NON-COAXIALITY

The Symmetry Principle predicts that asymmetric rock structure should be associated with non-coaxial rock flow, but it does not specify any particular mechanisms leading to this association, or admit of situations where the Principle can be violated. We take up these matters below.

The Symmetry Principle is apparently violated where flow proceeds with homogeneous (non-partitioned) velocity gradients and where the structural change is brought about by perfectly passive reorientation of structural elements. An approximation to this in rocks is the development of a grain-shape foliation by homogeneous simple shear of a set of initially equiaxed grains, with non-migrating boundaries. Here the resulting shape fabric, defined by the array of grain boundaries in the deformed state, displays the full symmetry of the strain component of the deformation (Fig. 7). We return to this apparent disobedience later.

When the Symmetry Principle is obeyed, one of the following descriptions applies.

Structural elements follow oblique special directions

Asymmetry of a structural assemblage arises where the total structure is made up of several elements and where these elements follow different, non-orthogonal, special directions in the bulk flow field. The best known example is provided by *S* and *C* foliation combinations in shear zones (Berthé *et al.*, 1979), where *S* approximately follows the flattening plane of finite strain and *C* approximately follows the internally non-rotating direction (eigendirection) that is parallel to the shear zone boundary. Other special directions that may influence the orientation of foliations and give asymmetry to total shear zone structures are the instantaneous elongation direction (Lister & Snoke 1984) and the second eigendirection (Bobyarchick 1986), or directions of maximum shear strain-rate (Simpson & De Paor 1992).

Separate parts of structures are introduced sequentially, in special directions

Here asymmetry arises, not because of simultaneous development of oblique elements, but because of sequential introduction of elements following a special direction with respect to which the material as a whole is rotating. The best example are curved crystal fibers developing as proposed by Durney & Ramsay (1973) and Wickham (1973). New fiber segments are introduced about parallel to the instantaneous elongation direction, while older ones are rotating out of this orientation. Another example is provided by the tips of propagating veins, which are aligned approximately in the local instantaneous shortening direction. It is necessary for this type of asymmetry that the older, rotated,

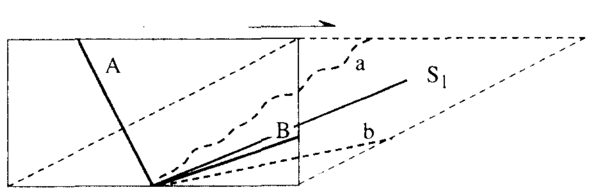


Fig. 8. Schematic diagram showing how veins a and b, symmetrically disposed across the maximum finite extension direction (S_1) in the deformed state, are expected to have somewhat different structures, if the early folding of vein a was incompletely reversed or removed as it rotated from the shortening field into the lengthening field (i.e. as it rotated through the normal to the shear direction).

parts of the structures be preserved as deformation proceeds.

Structural changes are imperfectly reversible

This reason for asymmetry is a very general one, and probably means that on *some* scale (perhaps very small) *all* rock fabrics developed during non-coaxial straining must be asymmetric. Even the grain-shape foliation mentioned above that was said to violate the Symmetry Principle, should, in a real rock, always be found to bear some slight asymmetry that would, after all, reveal its non-coaxial history.

The easiest example to understand of asymmetry resulting from irreversibility of structural change is provided by diversely oriented deformed vein sets in shear zones (Passchier 1990, Wallis 1992). When veins (or other strong layers) in orientations like A in Fig. 8 are sheared, they first shorten but later lengthen, if the shear strain is large enough. The shortening is accompanied by folding. The subsequent lengthening is accompanied by *partial* unfolding and pulling apart (vein a, Fig. 8). The structural change upon shortening is *incompletely reversed* upon lengthening. The vein therefore carries in its final state some evidence or structural 'memory' of its prior shortening. Contrast this with the history and structure of a vein in initial orientation B (Fig. 8). This vein ends up with the same net longitudinal strain as vein A, but it has never shortened, only elongated and come apart throughout the history. The resulting asymmetry of flow is manifest in the fact that veins a and b, equally-inclined to the finite flattening plane in the deformed state, have different shapes.

Another kind of strain reversal in shear zones, that is less familiar than the shortening-to-lengthening behavior above, is reversal in the instantaneous shear sense across material lines as they rotate through the stretching directions (Fig. 9, top). In a dextral shear zone, the shear sense changes from sinistral to dextral as lines rotate through the elongation stretching direction, and from dextral to sinistral as they rotate through the shortening stretching direction. On a Mohr circle for the velocity gradient tensor (see Passchier 1988), the rotation of material lines through the stretching directions is represented by circulation of points representing the lines around the circle (Fig. 9, bottom). The reversal in shear sense across a material line, with respect to its *instantaneously normal line*, can be seen from the fact

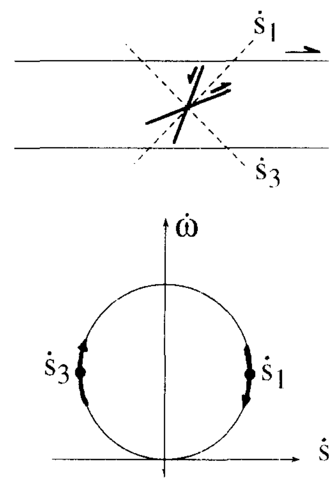


Fig. 9. Reversal of instantaneous shearing sense as material lines cross the stretching directions in a non-coaxial flow. Top part shows a material line (heavy) switching from sinistral to dextral shearing as it crosses the $\dot{s}_1$ stretching direction. The switch corresponds in the Mohr diagram below to the circulation (curved arrow) of a point representing the material line through the $\dot{s}_1$ point on the circle; past this point the clockwise angular velocity of the line becomes less than the clockwise angular velocity of an instantaneously orthogonal line, so the sense of shearing reverses.

that any point on the lower half of the circle corresponds to a material line with a lower clockwise angular velocity than its instantaneously normal line, represented by a point diametrically opposite on the upper half of the circle. Such a line is being sheared dextrally. The situation reverses when the point representing a line circulates into the upper half of the circle. Then the line begins to be sheared sinistrally. This reversal of shear sense of a line with respect to its *instantaneous* normal should not be confused with the reversal of finite shear strain of a line with respect to its *original* normal (discussed in Means 1976, p. 235, Simpson & De Paor 1992).

While shear structures with a memory of shear-reversal have not yet been described, such features may eventually be recognizable in rocks. Suppose for example that it becomes possible, perhaps by electron microscopy, to distinguish between twins in calcite that have sheared in one direction only from twins that have sheared first in the usual twinning direction and subsequently sheared in the opposite direction. (Reverse shearing across a twin is possible once the twin has some finite width. Reverse shearing untwins and narrows the lamella—Fig. 10.) If double-sense twins can be identified, then their orientations relative to the orientations of single-sense twins, and any associated finite strain indicator, should be useful for detecting and perhaps measuring non-coaxiality.

Structures mark patterns of flow perturbation, revealing the underlying asymmetry of flow

A criticism lodged against the kind of analysis above is that it relies too heavily on geometric properties of *bulk* flow and *bulk* deformation, overlooking the fact that deformation is conspicuously partitioned, on many scales (Bell 1981). This is certainly a legitimate criticism.

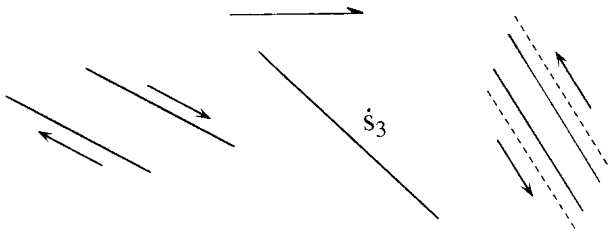


Fig. 10. Hypothetical reversal of shearing sense as a twin lamella rotates through the $\dot{s}_3$ stretching direction. The lamella achieves a certain width (left) under dextral shearing, then partially untwins and narrows (right) under sinistral shearing. The regions between the dashed and solid lines have twinned and then *untwinned* and may be microstructurally different from the regions outside the dashed lines, which have never been twinned.

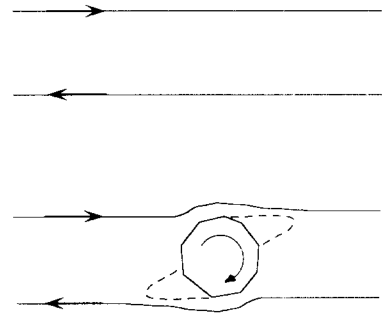


Fig. 11. Asymmetrical perturbation of a regular laminar flow (top) by growth within the material of a rigid porphyroblast. The resulting asymmetric structure comprises the deflected foliation around the porphyroblast and two pressure shadows (dashed). Movements are indicated with respect to a reference frame with a horizontal axis fixed to a material line midway between the horizontal arrows, and remote from the porphyroblast.

It cannot be correct, for example, to say that an *S* foliation in an *S* domain of an *S–C* deformed granite follows the finite flattening plane of the bulk deformation. The most the *S* foliation can do is follow the finite flattening plane for the *local* deformation within the *S* domain. A similar weakness exists in the suggestion made above that the sense of shear across a calcite twin lamella might reverse as the lamella rotated through a stretching direction. This was discussed as if the calcite grain was deforming homogeneously with the bulk flow. But most of a twinning calcite grain is in fact a rotating rigid object (assuming no deformation by slip). The current host and current twin lamellae are regions of zero strain-rate. The only actively straining regions at any given moment are narrow zones along each boundary of a broadening or narrowing twin lamella. So it makes little sense to treat the situation as if reversal in the bulk shear sense across a lamellae would necessarily lead to a reversal in the local shear sense. Whether and when this occurs across an individual lamellae depends on the unknown local displacement geometry around the lamella, or in other words on the direction and magnitude of the *local* shear stress. The only justification for using theoretical arguments based on the bulk flow geometry to predict local flow patterns and structural adjustments is that it is the best method we have at present, and it often seems to provide a pretty good approximation to the 'statistical' or averaged properties of fabrics. However it is always good to define bulk flow properties on as local a scale as possible—for example to relate fabric features in *S–C* granites, to a bulk flow in the *S* domains or to the different bulk flow in the *C* domains, rather than to one bulk flow for the shear zones as a whole. Used this way, continuum concepts can continue to be useful even in rocks with discontinuities in strain or rotation. But there are limits beyond which this approach becomes very difficult. These are met with for example when one considers asymmetric structural features developed in deforming blocky suspensions, such as the grain *tiling* structures described in granite by Blumenfeld & Bouchez (1988). Here the structural feature arises from piling-up of rotating rigid crystals, and it exists on approximately the same scale as the crystals themselves. Behavior like this, in particulate systems where the particle size is comparable with the scale of the structural features, is beyond the scope of this paper, except for the relatively simple case described below.

Some very familiar kinds of asymmetric structures in zones of non-coaxial flow arise because of flow perturbations induced in a matrix material around less deformable objects. Imagine for example, a mylonite flowing by some close approximation to dextral laminar flow. Then imagine that a rigid garnet porphyroblast begins to grow and perturb the laminar flow. Whatever the details of this perturbation, the local flow pattern around the upper left and lower right boundaries of the garnet are bound to be different from the local flow pattern around the lower left and upper right boundaries (Ghosh & Ramberg 1976). Many asymmetrical microstructures in shear zones arise from asymmetrical flow patterns of this general type, the best known example being asymmetric 'pressure shadows' enriched in quartz around strong objects like garnets (Fig. 11).

CONCLUDING REMARK

One may well ask whether structural geologists really need to understand such previously obscure rotational quantities as the spin and the internal vorticity. Are these quantities genuinely useful, or just trendy? My suggestion is that the spin and the internal vorticity, like the strain-rate, are part of the best-available framework we have for rigorous thinking about the kinematic significance of structure. Current and difficult problems, like the problem of porphyroblast rotation, must be approached by workers who are familiar with these parts of the terminology and techniques of continuum mechanics. On the other hand, it must be kept in mind that the spin, the internal vorticity and the strain-rate are all defined in continuum mechanics as mere *point properties*. By themselves, or in simple combinations like the vorticity number, they are not closely linked to structure, because most structure of interest (like a porphyroblast with spiral inclusion trails) extends over a finite volume of material and reflects the *distribution* of the spin, or internal vorticity, or strain-rate over the volume, and the variation of these *fields* and their *discontinuities* over time. So understanding spin and internal vorticity, or estimating bulk vorticity numbers for rocks, gets one started, but not very far toward understanding structure kinematically. Here, as in other parts of geological mechanics, the time seems ripe for a new emphasis on teaching theory relevant to structural geology, first through new literature such as the recent, focused teaching books by Passchier *et al.* (1990), Hanmer & Passchier (1991) and Bayly (1992), and eventually through generally strengthened university courses and curricula.

Acknowledgements—I thank Mike Willams, Sue Treagus, Scott Paterson and an anonymous reviewer for helping me with this paper. NSF grants for experimental work, currently grant EAR9204781, have given me the impetus and opportunity to improve my own understanding of this subject.

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